

Plasmonic wormholes: Defeating the early bird

Muamer Kadic,¹ Guillaume Dupont,¹ Sebastien Guenneau,¹ and Stefan Enoch¹

¹*Institut Fresnel, CNRS, Aix-Marseille Université,
Campus universitaire de Saint-Jérôme, 13013 Marseille, France*

(Dated: January 12, 2013)

We describe two types of toroidal metamaterials which are invisible for surface plasmon polaritons (SPPs) propagating on a metal surface. The former is a toroidal handlebody bridging remote holes on the surface: it essentially works as a plasmonic counterpart of electromagnetic wormholes. The latter is a toroidal ring lying on the metal surface: this bridges two disconnected metal surfaces i.e. it connects a thin metal cylinder to a flat metal surface with a hole. Full-wave numerical simulations demonstrate that an electromagnetic field propagating inside these metamaterials does not disturb the propagation of SPPs at the metal surface. A multilayered design of these devices is proposed, based on effective medium theory for a set of reduced parameters: The former metamaterial requires homogeneous isotropic magnetic layers, while the latter requires dielectric layers.

PACS numbers:

INTRODUCTION

Five years ago, two groups of physicists [1, 2] unveiled independent paths towards electromagnetic invisibility. The transformational optics proposal by Pendry et al. leads to singular tensors on the frontier of the invisibility region [3, 4] that require an extreme electromagnetic response achieved upon resonance of split ring resonators [5]. Various extensions including the blowup of a segment instead of a point [6] and the stereographic projection of a virtual hyper-sphere in a four dimensional space [7] have been studied. The conformal optics proposal by Leonhardt's grouping [2] leads to spatially varying, but bounded, scalar permittivity and permeability. However, the mathematical tools of complex analysis thus far constrain the invisibility design to two-dimensions. Some recent advances in quasi-conformal optics [8–11] also found some applications in the control of surface plasmon polaritons (SPPs): Transformational plasmonics [12–18]. Harnessing SPPs in order to deliver coupling between surface electrons on a structured metallic plate and incident light is a core topic in plasmonics [19–22], and plasmonic resonances underpins invisibility relying upon devices such as out-of-phase polarizability shells with low refractive index [23], core-shell anomalous resonances [24], or concentric rings of point scatterers [25]. However, the field of transformational plasmonics has a broader range of applicability as it is fuelled by analogies with Einstein's general relativity such electromagnetic wave propagation in inhomogeneous media and particle/light motion in gravitational potentials. For example, the plasmonic Eaton lens proposed by Zhang's team [18] is reminiscent of an optical black hole [26–28] which can trap and absorb electromagnetic waves coming from all directions. In the present letter, we extend the design of transformation based wormholes to the area of surface plasmon polaritons (SPPs). In physics and fiction, a wormhole is portrayed as a shortcut through

spacetime. Building upon the recent proposal of electromagnetic wormholes Greenleaf et al. [29], it is enough to consider it as a topological feature of space. Our main contribution is an explicit design of a toroidal handlebody to control SPPs propagating at a metal surface with two holes: The main ingredient in the wormhole recipe is to blow up a curve, rather than a point as is used in a typical three-dimensional invisibility cloak. We further numerically demonstrate the validity of our theoretical approach with three-dimensional finite element computations for SPPs propagating in a toroidal heterogeneous anisotropic wormhole. We finally derive some reduced set of parameters allowing for the design of a multi-layered toroidal tunnel consisting of an alternation of isotropic homogeneous layers approximating the ideal wormhole in the homogenization limit. This brings plasmonic wormholes a step closer to an experimental setup. Potential applications in plasmonics range from invisible Plasmonic waveguides (which could be useful in making measurements of electromagnetic fields without disturbing them, or as new types of endoscopes in medical applications), to hard discs for optical computers (the latter requires a tilted version of the wormhole which lies on a metal surface, which is also discussed). Other applications can be also envisaged thanks to the unprecedented control of SPPs through transformational plasmonics.

TOPOLOGICAL DESCRIPTION OF THE WORMHOLES

In this letter, we introduce two types of plasmonic wormholes. We start by describing the mathematical construction of a magnetic wormhole, which involves an invisible handlebody and two holes on a metal plate. Such a wormhole can be implemented for electromagnetic fields by deriving the required tensors of permittivity and permeability for a toroidal region of \mathbb{R}^3 (the invisible tun-

nel) connecting two regions of a metal surface, using the tools of transformational plasmonics.

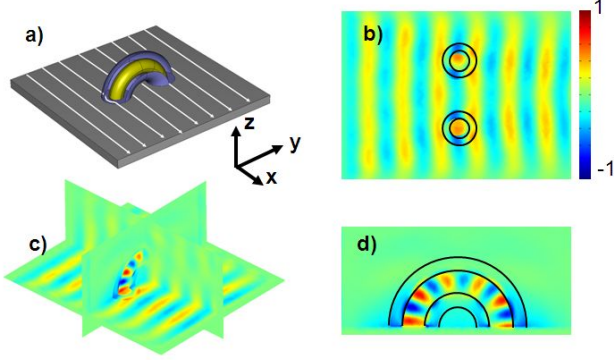


FIG. 1: Principle of the undetectable tunnel bridging two distant regions on a metal surface. The electromagnetic waveguide is shown in yellow and the coating in blue. It is coated with a transformed dielectric plasmonic wormhole with reduced parameters: in principle, the handlebody is acting as a waveguide for electromagnetic waves launched from the metal dielectric interface where we set $\mathbf{H} = (H_{x2}, 0, 0) \exp(-i\omega t)$ with $H_{x2} = \exp(-i\sqrt{2}z)$ (a); (b) Two dimensional plot (view from above) of the real part of the magnetic field; (c) Three-dimensional plot validating the guiding and invisibility properties; (d) Two-dimensional plot of the real part of the magnetic field in the vertical plane showing the inner structure of the wormhole with a dielectric in the middle region which is surrounded by two regions of transformed medium. Note that the interfaces between the regions consist of a thin, infinitely conducting layer of thickness 70 nanometers. These computations are for a wavelength of 700 nanometers for two remote circular holes of diameter $2a = 533$ nanometers with a center to center spacing of $2R = 1000$ nanometers.

The main ingredients of our wormhole construction are as follows: We start by making two identical holes in a metal plate, for instance two discs D_1 and D_2 separated by some positive distance on a plane. We denote by M the region so obtained: $M = \mathbb{R}^2 \setminus (D_1 \cup D_2)$. Topologically, M is a two-dimensional manifold with boundary, the boundary of M being $\partial M = \partial D_1 \cup \partial D_2$. We note that ∂M is the disjoint union of two discs on the plane.

The second component of the wormhole W is a curved toroidal cylinder, $T = \partial M \times [0; L]$, where L denotes the arc-length which connects points of circle ∂D_1 to points of circle ∂D_2 . As the boundaries of M and T are topologically the same ($\partial M = \partial T = \partial D_1 \cup \partial D_2$), we can glue these boundaries together. The resulting domain W no longer lies on the metal surface \mathbb{R}^2 , but rather has the topology of Euclidian space \mathbb{R}^3 with a three-dimensional handle attached, see Fig. 1(a). W is in fact a three-dimensional manifold (without boundary) that is the connected sum of the components M (metal surface with two distant holes) and T (toroidal cylinder).

Regarding the construction of the dielectric wormhole,

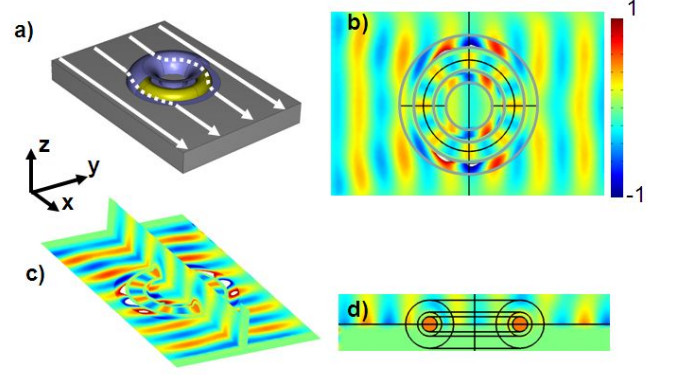


FIG. 2: (a) Principle of the undetectable toroidal ring bridging two disconnected regions i.e. a metal cylinder and a metal surface. The electromagnetic waveguide is shown in yellow and the coating in blue. Ray trajectories are drawn for illustrative purpose only. Full wave simulations validate the theoretical proposal: (b) View from top; (c) 3D plot; (d) Side view; Here, all plots are for the real part of the magnetic field and we set $\mathbf{H} = (0, 0, H_{z2}) \exp(-i\omega t)$ with $H_{x2} = \exp(-i\sqrt{2}z)$ on the waveguide cross-section in the vertical plane $y = 0$.

see Fig. 2, the previous construction repeats mutatis mutandis with the noticeable difference that the manifold M should be replaced by a manifold M_1 with a single hole: $M_1 = \mathbb{R}^2 \setminus D_1$. Moreover, the disc D_2 is now located inside D_1 : $D_2 \subset D_1$. Further details can be found in the Supplementary Information.

TRANSFORMATION PLASMONICS FOR THE DESIGN OF WORMHOLES

We now wish to apply tools of transformational plasmonics to design a device in \mathbb{R}^3 which controls the propagation of SPPs in the same way as the presence of the handle T in the wormhole manifold W . On W we shall use the Riemannian metric that is the Euclidian metric on M and the product metric on T . We emphasize that we are not actually tearing and gluing plasmonic space, but instead prescribing a metamaterial which makes the SPPs propagating on the metal plate behave as if they were propagating on the wormhole manifold W . In other words, adopting the viewpoint of a SPP, it appears that the topology of plasmonic space has been changed.

We first consider a surface plasmon polariton (SPP) propagating in the positive x direction at the interface $z = 0$ between a metal surface ($z < 0$) and air ($z > 0$). If we choose the magnetic field \mathbf{H} as the unknown, it takes the following form:

$$\begin{cases} \mathbf{H}_2 = (0, H_{y2}, 0) \exp\{i(k_{x2}x - \omega t) - k_{z2}z\}, & z > 0, \\ \mathbf{H}_1 = (0, H_{y1}, 0) \exp\{i(k_{x1}x - \omega t) + k_{z1}z\}, & z < 0, \end{cases} \quad (1)$$

with $\Re(k_{z1})$ and $\Re(k_{z1})$ strictly positive in order to maintain evanescent fields above and below the interface $z = 0$.

Let us now consider two holes in the metal interface. It is clear that the propagation of the SPP is affected by their presence (see supplementary information). Our aim is to design an invisible handlebody through geometric which will bridge the two holes at the metal surface, and make them invisible.

For simplicity, we construct a device that has rotational symmetry about a line in \mathbb{R}^3 , and moreover we assume that the radii of D_1 and D_2 are equal. We use toroidal coordinates (r, u, v) corresponding to a point $((R+r \sin u) \cos v, (R+r \sin u) \sin v, r \cos u)$ in \mathbb{R}^3 , where $2R$ is the center-to-center spacing between the discs D_1 and D_2 . We then blowup the centerline of the toroid which goes through the centers of D_1 and D_2 , onto a toroidal coating using the transform $r' = a + r(b-a)/b$, $u' = u$ and $v' = v$. Here, b and a are the radii of the circles that form the outer and inner boundaries of the cloaking region, respectively.

In order to simplify the design of the wormhole, we proceed in a way similar to what was done to obtain a reduced set of material parameters for cylindrical cloaks in [30]. Using the transformational plasmonics tools, we obtain (see online Supplementary Information):

$$\begin{aligned} \varepsilon_{rr} &= \mu_{rr} = \frac{r-a}{r} f(r, u) \\ \varepsilon_{uu} &= \mu_{uu} = \frac{r}{r-a} f(r, u) \\ \varepsilon_{vv} &= \mu_{vv} = \frac{r-a}{(b-a)^2} \frac{r}{r} f(r, u) \end{aligned} \quad (2)$$

where $f(r, u) = \frac{(b-a)R+b(r-a)\sin u}{(b-a)(R+r\sin u)}$, which leads us to the set of reduced parameters

$$\begin{aligned} \varepsilon_{rr} &= \mu_{rr} = \left(\frac{r-a}{r} \right)^2 \\ \varepsilon_{uu} &= \mu_{uu} = 1 \\ \varepsilon_{vv} &= \mu_{vv} = \frac{b^2}{(b-a)^2} \end{aligned} \quad (3)$$

that preserve the wave trajectories, but induce a slight impedance mismatch on the wormhole outer boundary.

For a toroidal cloak, see Fig. 2(a), both angles u and v vary in the range $(0, 2\pi)$. However, for a plasmonic wormhole, see Figure 1(a), we only need the upper half of a toroidal cloak, that is u varies between 0 and π . If on the contrary we concentrate on the toroidal cloak lying on the metal surface, see Fig. 2, we now need to tilt the toroidal cloak by an angle of $\pi/2$ and further cut it in two halves along the z -axis, so that it is now v which varies from 0 to π .

We note that if the toroidal cloak component of the wormhole consists of an alternation of two homogeneous isotropic layers of thicknesses d_A and d_B and permittivity

$\varepsilon_A = \lambda_A^{-1}$, $\varepsilon_B = \lambda_B^{-1}$ and permeability $\mu_A = \lambda_A$ and $\mu_B = \lambda_B$, we have

$$\frac{1}{\lambda_{rr}} = \frac{1}{1+\eta} \left(\frac{1}{\lambda_A} + \frac{\eta}{\lambda_B} \right), \quad \lambda_{uu} = \lambda_{vv} = \frac{\lambda_A + \eta \lambda_B}{1+\eta} \quad (4)$$

where $\eta = d_B/d_A$ is the ratio of thicknesses for layers A and B and $d_A + d_B = 1$.

We report in Fig. 3 some computations for a SPP incident at 700 nanometers upon the structured magnetic wormhole consisting of an alternation of homogeneous magnetic layers specified in the online Supplementary Information. If we now tilt the wormhole by an angle $\pi/2$, a similar design holds with an alternation of dielectric homogeneous layers, see Fig. 4. Further technical details, as well as numerical results obtained for 800 nanometers, can be found in the online supplementary information.

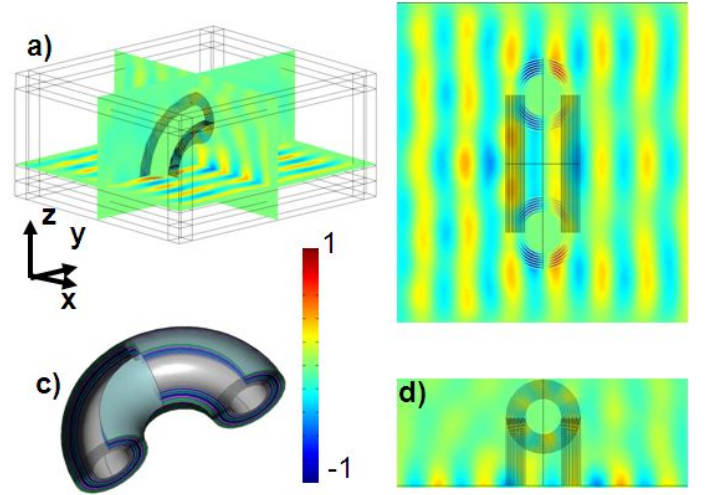


FIG. 3: Structured magnetic wormhole: SPP Gaussian beam with a waist of 2100 nanometers incident upon a multi-layered cylindrical wormhole at 700 nanometers smoothly bent around a metal toroidal obstacle; The permeability in every layer (of identical thickness 60nm) is given by $\mu_i = [0.0032; 7.9796; 0.0467; 7.9178; 0.1205; 7.8363; 0.2066; 7.7476; 0.2965; 7.6575; 0.386; 7.5691; 0.4729; 7.4839]$, (from the inner to the outer layer); (a) Three-dimensional plot of the real part of the magnetic field; (b) Top view; (c) Diagrammatic view of the device; (d) Side view. The color scale has been normalized.

It is enlightening to look at the dispersion relation for the surface plasmon at the interface between the metal surface and the handlebody. The latter is described by diagonal tensors of relative permittivity and permeability $\underline{\varepsilon}_2 = \text{diag}(\varepsilon_{xx2}, \varepsilon_{yy2}, \varepsilon_{zz2})$ $\underline{\mu}_2 = \text{diag}(\mu_{xx2}, \mu_{yy2}, \mu_{zz2})$ as v and z coincide at $z = 0$. The boundary condition at the interface $z = 0$ requires continuity of the tangential components of the electromagnetic field, which brings

$$\frac{k_{z1}}{\varepsilon_1} + \frac{k_{z2}}{\varepsilon_{xx2}} = 0. \quad (5)$$

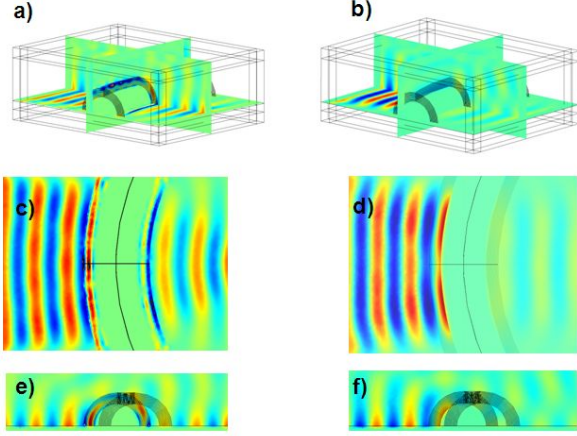


FIG. 4: Structured dielectric wormhole: SPP Gaussian beam with a waist of 2100 nanometers incident at 700 nanometers upon a metal toroidal obstacle dressed with the wormhole (left panel) and on its own (right panel). The Permittivity in every layer (of identical thickness 60nm) is given by: $\epsilon_i = [0, 0032; 7, 9796; 0, 0467; 7, 9178; 0, 1205; 7, 8363; 0, 2066; 7, 7476; 0, 2965; 7, 6575; 0, 386; 7, 5691; 0, 4729; 7, 4839]$; (a,b) Three-dimensional plot; (c,d) View from above; (e,f) Side view. The color scale has been normalized

The existence of a surface plasmon thus requires ϵ_1 and ϵ_{xx2} to be of opposite sign. Importantly, no such requirement is needed for the permeability, i.e. the transformed metal need not be magnetic. From (5), we deduce the dispersion relation for the SPP:

$$k_x = \frac{\omega}{c} \sqrt{\frac{\epsilon_{zz2}\epsilon_1(\mu_{yy}2\epsilon_1 - \epsilon_{xx2})}{\epsilon_1\epsilon_1 - \epsilon_{xx2}\epsilon_{zz2}}}. \quad (6)$$

CONCLUSION

In conclusion, we have studied analytically and numerically the extension of wormholes to the domain of surface plasmon waves propagating at the interface between metal and dielectric/air. These waves obey the Maxwell equations at a flat interface and are evanescent in the transverse direction, so that, the problem we have treated is somewhat a two-dimensional version of the wormhole design by Greenleaf et al. [29]: it is enough to consider the wormhole as a manifold in a three dimensional Euclidean space. Nevertheless, our numerical computations based on the finite element method take into account the three dimensional features of the problem, such as plasmon polarization and jump of permittivity at the interface between metal and metamaterial/air which are described by permittivities of opposite sign.

Wormholes introduced by Greenleaf et al. represent a fascinating electromagnetic paradigm, but were initially

thought as an abstract metamaterial bridging two spherical holes, thereby requiring a further dimension for the invisible tunnel, and moreover no permeability and permeability tensors were derived for a specific design. In the present letter, we have transposed this idea to the area of surface plasmon polaritons, with two illustrative examples of invisible handlebody and ring over surfaces, and further proposed a multi-layered version of these metamaterials to foster applications in the emerging field of transformational plasmonics.

-
- [1] J.B. Pendry, D. Schurig, and D.R. Smith, *Science* **312**, 1780 (2006).
 - [2] U. Leonhardt, *Science* **312**, 1777 (2006).
 - [3] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, *Communications in Mathematical Physics* **275**(3), 749-789 (2007).
 - [4] R. V. Kohn, H. Shen, M. S. Vogelius, and M. I. Weinstein. Cloaking via change of variables in electric impedance tomography. *Inverse Problems* **24**, 015016 (2008).
 - [5] D. Schurig, J.J. Mock, B.J. Justice, S.A. Cummer, J.B. Pendry, A.F. Starr, D.R. Smith, *Science* **314**, 977 (2006).
 - [6] W.X. Jiang et al. "Invisibility cloak without singularity," *Appl. Phys. Lett.* **93**, 194102 (2008).
 - [7] U. Leonhardt and T. Tyc, "Broadband invisibility by non-euclidean cloaking," *Science* **323**, 110 (2009).
 - [8] J. Li and J.B. Pendry, "Hiding under the carpet: a new strategy for cloaking," *Phys. Rev. Lett.* **101**, 203901 (2008).
 - [9] R. Liu, C. Ji, J.J. Mock, J.Y. Chin, T.J. Cui and D.R. Smith, "Broadband Ground-Plane Cloak," *Science* **323**, 366 (2008).
 - [10] J. Valentine, J. Li, T. Zentgraf, G. Bartal and X. Zhang, "An optical cloak made of dielectrics." *Nature Mater.* **8**, 569 (2009).
 - [11] L. H. Gabrielli, J. Cardenas, C. B. Poitras, and M. Lipson, Silicon nanostructure cloak operating at optical frequencies, *Nat. Photonics* **3**(8), 461463 (2009).
 - [12] I. I. Smolyaninov, Transformational optics of plasmonic metamaterials, *New J. Phys.* **10**(11), 115033 (2008).
 - [13] P. A. Huidobro, M. L. Nesterov, L. Martin-Moreno, F. J. Garcia-Vidal, "Transformation optics for plasmonics" *Nano Lett.* **10** 198590, (2010).
 - [14] M. Bashevoy, V. Fedotov, and N. Zheludev, "Optical whirlpool on an absorbing metallic nanoparticle", *Opt. Express* **13**, 8372-8379 (2005)
 - [15] Y. Liu, T. Zentgraf, G. Bartal, X. Zhang, "Transformational plasmonics" *Nano Lett.* **10** 19917, (2010).
 - [16] M. Kadic, S. Guenneau, S. Enoch, "Transformational plasmonics: cloak, concentrator and rotator", *Opt. Express* **18** 1202732, (2010).
 - [17] J. Renger, M. Kadic, G. Dupont, S. Acimovic, S. Guenneau, R. Quidant, S. Enoch, "Hidden progress: broadband plasmonic invisibility" *Opt. Express* **18** 15757-68, (2010).
 - [18] T. Zentgraf, Y. Liu, M.H. Mikkelsen, J. Valentine and X. Zhang, Plasmonic Luneburg and Eaton lenses, *Nature Nanotechnology* (DOI: 10.1038/NNANO.2010.282), (2010).

- [19] W.L. Barnes, A. Dereux, T.W. Ebbesen Surface plasmon subwavelength optics, *Nature* **424**, 824-830 (2003).
- [20] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, P. A. Woff, *Nature* **391**, 667 (1998).
- [21] J.B. Pendry, L. Martin-Moreno and F.J. Garcia-Vidal, Mimicking surface plasmons with structured surfaces, *Science* **305**, 847
- [22] F.J. Garcia de Abajo, G. Gomez-Santos, L.A. Blanco, A.G. Borisov and S.V. Shabanov, "Tunneling mechanism of light transmission through metallic films," *Physical Review Letters* **95** 067403 (2005).
- [23] A. Alu and N. Engheta, "Achieving transparency with plasmonic and metamaterial coatings," *Phys. Rev. E* **72** 016623 (2005). (2004
- [24] N.A. Nicorovici, R.C. McPhedran and G.W. Milton, "Optical and dielectric properties of partially resonant composites," *Phys. Rev. B* **49**, 8479-8482 (1994).
- [25] B. Baumeier, T.A. Leskova and A.A. Maradudin, "Cloaking from surface plasmon polaritons by a circular array of point scatterers," *Physical Review Letters* **103**, 246809 (2009).
- [26] Q. Cheng, T.J. Cui, W.X. Jiang and B.G. Cai, An omnidirectional electromagnetic absorber made of metamaterials, *New J. Phys.* **12**, 063006 (2010).
- [27] E. E. Narimanov, A. V. Kildishev, "Optical black hole: Broadband omnidirectional light absorber," *Appl. Phys. Lett.* **95**, 041106 (2009).
- [28] D. A. Genov, S. Zhang, X. Zhang, "Mimicking celestial mechanics in metamaterials," *Nature Phys.* **5**, 691, (2009).
- [29] A. Greenleaf, Y. Kurylev, M. Lassas, G. Uhlmann: "Electromagnetic wormholes and virtual magnetic monopoles from metamaterials," *Physical Review Letters* **99**, 183901 (2007).
- [30] W. Cai, U.K. Chettiar, A.V. Kildiev and V.M. Shalaev, "Optical Cloaking with metamaterials", *Nature Photonics* **1**, 224-227 (2007).